

Integration multiple procedures

Compute the following integrals:

$$1. \int \frac{8dx}{x^3 - 4x}; \quad \int \frac{5x^2 - 9}{x^3 - 9x} dx; \quad \int \frac{x+2}{x^4 + 2x^3 + x^2}; \quad \int \frac{6x^2 + 3x + 4}{x^3 + 2x} dx;$$

$$2. \int \frac{4x^3 + 3x^2 + 18x + 12}{(x^2 + 4)^2} dx; \quad \int_0^{\pi/4} e^x (\sqrt{3} \cos 2x - \sin 2x) dx; \quad \int_0^1 (x^2 e^x + \frac{x^2}{2} - \ln(1+x)) dx;$$

$$3. \int \frac{5x^2 + 12x + 9}{x^3 + 3x^2 + 3x} dx; \quad \int \frac{5x+9}{(x-9)x^{3/2}} dx; \quad \int \frac{\sqrt{x}}{x^3 + 2x^2 - 3x} dx; \quad \int \frac{dx}{x - x^{4/3}};$$

$$4. \int_0^3 \frac{dx}{(x+2)\sqrt{x+1}}; \quad \int_0^{1/2} \frac{dt}{\sqrt{2t}(9 + \sqrt[3]{2t})}; \quad \int_3^{29} \frac{(t-2)^{2/3}}{(t-2)^{2/3} + 3} dt; \quad \int x^5 \sqrt{1+x^3} dx;$$

$$5. \int \frac{x^5}{(2+3x^3)^{3/2}} dx; \quad \int \frac{dx}{x^3(1+x^3)^{1/3}}; \quad \int \frac{dx}{1 + \sin x + \cos x}; \quad \int \frac{dx}{\sin x + \operatorname{tg} x};$$

$$6. \int \frac{dx}{5+4\cos x}; \quad \int \frac{dx}{1+\sin x - \cos x}; \quad \int_0^{\pi} \frac{dx}{5-3\cos x}; \quad \int_0^{\pi/2} \frac{dx}{2+\cos x};$$

$$7. \int_0^{\pi/2} \frac{dx}{2+\sin x}; \quad \int_0^{\pi/6} \frac{1+\operatorname{tg} x}{1+\sin 2x} dx; \quad \int_0^1 \frac{x-2}{(2x-3)^2} dx; \quad \int_0^1 \frac{e^x - 1}{e^x + 1} dx;$$

8. Verify the following:

$$\int_0^1 \frac{dx}{e^x + e^{-x}} = \operatorname{arctg} e - \frac{\pi}{4}, \text{ (hint : } e^x = z \text{);} \quad \int_0^a \frac{dx}{\sqrt{ax - x^2}}, \text{ (hint : } x = a \sin^2 z \text{)}$$

$$\int_0^1 \sqrt{2t+t^2} dt = \sqrt{3} - \frac{1}{2} \ln(2+\sqrt{3}), \text{ (hint : } t+1=z \text{);} \quad \int_{1/3}^1 \frac{(x-x^3)^{1/3}}{x^4} = 6, \text{ (hint : } x = \frac{1}{z} \text{)}$$

$$\int_0^1 \frac{x^3}{(1+x^2)^{3/2}} dx; \text{ Deduce the limit value of the following sum } S_n = \frac{1}{n^4} \sum_{k=1}^{k=n} \frac{k^3}{\sqrt{\left(1 + \frac{k^2}{n^2}\right)^3}}$$