1. Study the nature of the following integrals:

a) $\int_0^1 \frac{Logt}{(1+t)^2}dt$

b) $\int_0^1 \frac{1}{e^x-\cos x}dx$

c) $\int_{-1}^0 t^2 Log|t|dt$

d) $\int_0^1 \frac{1}{t\sqrt{1-t^2}}dt$

e) $\int_{-1}^0 \frac{e^t}{\sqrt{1-e^t}}dt$

f) $\int_0^1 \frac{Logx}{\sqrt{x}}dx$

g) $\int_0^1 \frac{x}{(1+x^2)\sqrt{1-x^4}}dx$

h) $\int_1^{+\infty} \frac{\cos x}{\sqrt{x}}dx$

i) $\int_1^{+\infty} \frac{1}{t}(e^{\frac{1}{t}}-\cos\frac{1}{t})dt$

j) $\int_{-\infty}^{+\infty} \frac{t^7}{t^{16}+1}dt$

k) $\int_e^{+\infty} \frac{dt}{t\ln t\ln(\ln t)}$

l) $\int_0^{+\infty} \frac{\sin\frac{1}{x^2}}{Log(1+\sqrt{x})}dx$

m) $\int_1^{+\infty} Logxdx$

n) $\int_1^{+\infty} \frac{\text{Arctg}(x-1)}{(x^2-1)^{\frac{4}{3}}}dx$

o) $\int_0^1 \sin\frac{1}{t}e^{-\frac{1}{t}}t^k dt;\ k\in N$

p) $\int_0^{+\infty} t^\alpha \ln(1+t)dt;\ \alpha\in R$

q) $\int_0^{+\infty} e^{-\alpha x}x^\beta dx;\ (\alpha,\beta)\in R^2$

2. Let K be a real number; show that $\int_1^{+\infty} t^{k-1}\cos t dt\quad k<0$ is convergent.

a) Deduce that $\int_1^{+\infty} t^k \sin t dt\ \ k<0$ is defined.

b) Use the precedent result to study $\int_1^{+\infty} \sqrt{t}\sin(t^2)\,dt$

3. Show that $\int_{-\infty}^{+\infty} e^{(x-e^x)}dx$ is convergent and evaluate its value.

4. find the surface bounded by these curves and their asymptotes:

$$y^2=\frac{x^2}{1-x^2};\quad y=\frac{8}{x^2+4};\quad y=\frac{x}{(4+x^2)^2};\quad y=xe^{\frac{x^2}{2}}$$

5. Let: $I=\int_0^1 \frac{Logx}{1+x^2}dx$ and $J=\int_1^{+\infty}\frac{Logx}{1+x^2}dx$

a) Show that they are convergent and that I+J=0

b) Deduce that $K=\int_0^{+\infty}\frac{Logx}{a^2+x^2}dx=\frac{\pi}{2a}Loga$ *(variable substitution)*

6. Let's consider the following integral: $\Gamma(\alpha)=\int_0^{+\infty} x^{\alpha-1}e^{-x}dx$

a) Study its convergence.

b) Integrate by part and find a relation between $\Gamma(\alpha)$ and $\Gamma(\alpha+1)$

c) For $\alpha \in N^*$, give $\Gamma(\alpha+1)$ as a function of α.

7. study convergence of:

$$B(p,q)=\int_0^1 x^{p-1}(1-x)^{q-1}dx \quad (p,q)\in R^2$$

a) Show that B(p, q) = B(q, p)

b) Prove that $B(p,q)=2\int_0^{\pi/2}(\sin x)^{2p-1}(\cos x)^{2q-1}dx$

c) Show that $I=\int_0^{+\infty} e^{-ax}\cos bx dx$ and $J=\int_0^{+\infty} e^{-ax}\sin bx dx$ where $a>0$ are absolutely convergent.

d) Evaluate I and J.

e) Deduce the value of: $K=\int_0^{+\infty} e^{-x}\sin^3 x dx$

8. Show that $\int_0^1 \sqrt{\frac{1-x}{x}}dx$ is convergent and evaluate its value.

9. Show that $I(p)=\int_0^{+\infty}\frac{Log(1+x)}{x^{p+1}}dx$ is convergent if 0<p<1. (ex: $I\left(\frac{1}{2}\right)=2\pi$)

10. Study the convergence of $\int_0^{+\infty}\frac{1}{(\cos^2 x-\cos^2\alpha)^m}dx \quad 0<\alpha<\pi/2$

11. For every integer n: $I_n=\int_0^1\frac{x^{2n+1}Logx}{x^2-1}dx$

a) Prove the existence of I_n .

b) Evaluate I_{n+1} -I_n.

c) Show that $\forall x\in\,]0,1[\Rightarrow 0<\frac{xLogx}{x^2-1}<1/2$. Deduce $\lim_{n\to+\infty} I_n=0$

12. Study the convergence of $\int_0^{+\infty}\sin x\sin\frac{1}{x}dx$.

13. Let f be a continuous function such that:

$$\lim_{x\to+\infty} f(x)=A \text{ and } \lim_{x\to-\infty} f(x)=B$$

a) Show that this integral $\int_{-\infty}^{+\infty}[f(x+a)-f(x)]dx$ exists and evaluate its value.