Part1

1. Find A+B, A-B, 3A, -2B in each of the following cases:

   - A=(2,-1), B=(-1,1) ; • A=(-1,3), B=(0,4)
   - A= (2,-1,5), B=(-1,1,1,) ; • A=(π,3,-1),B=(2π,-3,7)

2. Find the located vectors PQ and AB that are equivalent and parallel

   - P=(1,-1), Q=(4,3), A=(-1,5), B=(5,2)
   - P=(1,4), Q=(-3,5), A=(5,7), B=(9,6)
   - P=(1,-1,5), Q=(-2,3,-4), A=(3,1,1), B=(0,5,1)
   - P=(1,2,4), Q=(-1,3,5), A=(-2,3,-1), B=(-11,3,-28)

3. Compute $A\cdot A, A\cdot B$, $(A+B)^2$, $(A-B)^2$ when:

   - A=(2,-1,3), B=(-1,1,1) ; A=(1,-1,1), B=(2,1,5)
   - Are they perpendicular?

4. Demonstrate that if A is perpendicular to any vector X, means that A=0

5. Find in each of the following cases the norm of vector A, the projection of A over B, the cosine of their angle:

   - A=(1,-2), B=(5,3) • A= (-2,1,4), B=(-1,-1,3)
   - A=(-1,1,0), B=(2,1,-1) • A=(1,-2,3), B=(-3,1,5).

6. Determine the angles of the triangle formed by the following points: (2,-1,1), (1,-3,-5), (3,-4,-4)

7. Prove that:

   - $\|A+B\|^2 + \|A-B\|^2 = 2\|A\|^2 + 2\|B\|^2$
   - $\|A+B\|^2 - \|A-B\|^2 = 4AB$

8. Show using an opposite example that if A.B = A.C, B is not necessarily equal to C.

9. Write a parametric representation of the lines passing through the following points:

   - P=(1,3,-1), Q=(-4,1,2) • P=(-1,5,3), Q=(-2,4,7)
   - P=(1,1,-1), Q=(-2,1,3) • P=(-1,5,2), Q=(3,-4,1)

10. Find the equation of the plan passing though point P and normal to vector N:

- N=(1,-1,3) et P=(4,2,-1)
- N=(-3,-2,4) et P=(2,π,-5)
- N=(-1,0,5) et P=(2,3,7)
- N=(1,1,1) et P=(1,1,1)

11. find the equation of the line passing through (-5,3) and perpendicular to the vector (1,-1)

12. Find the distance between the point (1,1,2) and the plane 3x + y - 5z = 2, write the general formula that gives this distance.

Part 2

13. Find the velocity vector of the following curves:

- $(\cos t, \sin t); \quad 0 \le t \le 2\pi$
- $(e^t, \cos t, \sin t); \quad 0 \le t \le 2\pi$
- $(\sin 2t, Log(1+t), t); \quad 0 \le t \le 1$

Tell if the acceleration vector is perpendicular to the velocity vector

14. Let $X(t); t \in I$ be a differentiable curve. Find the equation of the line that is perpendicular to the curve $(\cos 3t, \sin 3t); \quad 0 \le t \le 2\pi$ when t= π/3, and the equation of the plane that is perpendicular to the curve $(e^t, t, t^2); t \in \Re$ at the points t = 1 et t = 0.

15. Let $X(t)$ be a differentiable curve defined in an open boundaries interval I. Let's consider a point Q that is not part of the curve.

- Write the formula giving the distance between Q and any point of the curve
- If t_o is the value for which the distance between Q and X(t_o) is minimum, prove that the vector Q-X(t_o) is normal to the curve at X(t_o).
- If X(t) is a line, prove that this t_o is unique.

16. Prove that if the velocity is constant, it is perpendicular to the acceleration vector

17. Prove that if the acceleration vector is always perpendicular to the velocity, the velocity is constant.

18. Let B be a non null vector and a curve $X(t); t \in I$ in such a way that X(t)**.**B = t for every t. Let's suppose that the angle between X'(t) and B is constant. Prove that X"(t) is always perpendicular à X'(t).

19. Write the parametric equation of the curve that is tangent to the following ones :

- $(\cos 4t, \sin 4t, t)$; $t \in \Re$ at point $t = \pi/8$
- $(t, 2t, t^2)$; $t \in \Re$ at point $A = (1,2,1)$
- $(e^{3t}, e^{-3t}, \sqrt{2t})$; $t \in \Re$ at point t=1
- (t, t^3, t^4); $t \in \Re$ at point t=(1,1,1)

20. Let X(t) = $\left(\frac{2t}{1+t^2}, \frac{1-t^2}{1+t^2}, 1\right)$; $t \in \Re$. Show that the angle between X(t) and X'(t) is constant.

21. Compute the length of the following arcs:

- (cos2t, sin2t, 3t); $t \in \Re$ between t=1 and t=3.
- (t-sint, 1-cost); $t \in \Re$ between t=0 and t=2π.
- (t, Logt); $t \in \Re$ between t=1 and t=2.

22. Demonstrate that the following two curves: $(e^t, e^{2t}, 1-e^{-t})$ and (1-t, cost, sint) ; $t \in \Re$ have an intersection at point (1,1,0). What the angle between their respective tangents at this point.

23. What are the intersection points between this curve $(2t^2, 1-t, 3+t^2)$; $t \in \Re$ and this plane $8x - 14y + z - 10 = 0$

24. Let X(t) = (acost, asint, bt); $t \in \Re$ with a and b constant. Let u(t) be the angle of the tangent at a given point of the z axis. Show that cos u(t) is constant and equals: $\frac{b}{\sqrt{a^2+b^2}}$.

25. Let B a unit vector, and X(t) ; $t \in \Re$ a curve such that $\forall t;\ X(t) * B = e^{2t}$. Let's suppose that the velocity vector of the curve makes a constant angle u with vector B and 0 < u < π/2.

- Show that the velocity v(t) equals $\frac{2e^{2t}}{\cos u}$
- Comoute the scalar product X'(t).X"(t) as a function of t and u.