

Part 1

1. Do the following functions have limits when (x, y) is close to $(0,0)$?

$$f(x,y) = \frac{x^2 - y^2}{x^2 + y^2}, \quad f(x,y) = \frac{|x+y|}{x^2 + y^2}, \quad f(x,y) = \frac{x^3 y^3}{x^2 + y^2}, \quad f(x,y) = \frac{xy^2}{x^2 + y^2};$$

$$f(x,y) = \frac{x^2 + y^3}{x^2 + y^4}$$

2. $f(x, y) = \frac{x^2 y}{x^4 + y^2}$. Study the limit of $f(x,y)$ at $(0, 0)$ when (x,y) approaches $(0,0)$ using the following paths

- a) A line $y = tx$, ($t = \text{cte}$)
- b) A parabola $y = x^2$
- c) What can you conclude?

3. Study the continuity at $(0,0)$ of the following functions:

a) $f(x,y) = \frac{(x+y)^2}{x^2 + y^2}$ if $(x, y) \neq (0,0)$ et $f(0,0) = 0$

b) $f(x,y) = (x^2 + y^2) \sin \frac{1}{x^2 + y^2}$ if $(x, y) \neq (0,0)$ et $f(0,0) = 0$

4. $f(x, y) = \frac{x^2}{x^2 + y^2}$. Compute the following limits:

a) $\lim_{y \rightarrow 0} [\lim_{x \rightarrow 0} f(x, y)]$; b) $\lim_{x \rightarrow 0} [\lim_{y \rightarrow 0} f(x, y)]$; c) $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$.

Part 2

5. Compute the first and second partial derivatives of the following functions :

$$xy + z, \quad x^2 y^5 + 5, \quad \sin xy + \cos yz, \quad e^{xyz}, \quad x^y, \quad \text{Log}(x + y)$$

6. $f(x,y) = xy \frac{x^2 - y^2}{x^2 + y^2}$ if $(x, y) \neq (0,0)$ et $f(0,0) = 0$

Compute $f''_{xy}(0,0)$ and $f''_{yx}(0,0)$. What can we deduce? Is f differentiable at point $(0,0)$?

7. Let $f(x, y) = g(x) + h(y)$. Let's assume that function g is differentiable in x_0 and h differentiable in y_0 . Demonstrate, using the definition, that f is differentiable in (x_0, y_0)