Part 1

1. Let f be a function that verifies grad f(1,1,1) = (5,2,1). Let $C(t) = (t^2, t^{-3}, t)$
   Compute $f_t'(C(t))$ at $t = 1$
2. Lets consider the following polar coordinates: x=rcosθ y= rsinθ .
   a) Compute: $r_x', r_y', \theta_x', \theta_y'$
   b) Show using an appropriate variable subtituion (polar coordinates) that every derivable function f(x,y) verifying the following identity:
   $$xf_x' + yf_y' = 0$$
   Can be transformed into a function of $\frac{y}{x}$.
3. If U = *f(x- y, y- x)*, show that: $\frac{\partial U}{\partial x} + \frac{\partial U}{\partial y} = 0$
4. Let $U(x, y, z) = F\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$ Show that: $x^2U_x' + y^2U_y' + z^2U_z' = 0$
5. Let *g(x, y)= f(x+ y, x- y)*, where v is a two variables, u & v, differentiable fucntion. Show that:
   $$\frac{\partial g}{\partial x}\frac{\partial g}{\partial y} = \left(\frac{\partial f}{\partial u}\right)^2 - \left(\frac{\partial f}{\partial v}\right)^2$$
6. Using the following variable substitution *u=xy* et *v=x/y*, find the functions *z(x, y)* that satisfies : $xz_x' + yz_y' = 2xy$
7. Find all the functions of class C^2, defined on R_*^{2+} such that:
   $$x^2\frac{\partial^2 f}{\partial x^2} - y^2\frac{\partial^2 f}{\partial y^2} = 0$$

We can use the following variable substituion *v= x/y; u= xy*

8. Le Laplacien dune fonction est défini par $\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$. Une fonction est dite harmonique si son Laplacien est nul
   c) Montrer que le Laplacien d'une fonction à deux variables s'écrit en coordonnées polaires:
   $$\Delta u = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}$$
   d) Trouver la fonction harmonique la plus générale quand u est fonction de r seulement.
   e) Même question si $u = r^n f(\theta)$, n étant un entier naturel.

Part 2

9. Let $f(x,y,z) = z - e^{x}\sin(y)$ and $P = \left(Log(3), \frac{3\pi}{2}, -3\right)$. Compute:

   a) The directional derivative o f at point P in the direction u = (1, 2, 2)

   b) The maximum and the minimum of the directional derivative of f at point P.

10. Find the directional derivative for the following functions:

    a) $f(x,y) = Log\left(\sqrt{x^2 + y^2}\right)$; P = (1,1) ; u = (2,1)

    b) $f(x,y.z) = xy + yz + zx$; P = (-1,1,7) ; u = (3,4,-12)

    c) $f(x,y.z) = 4x^2 + 9y^2$; P = (2,1) in the direction of the maximum directional derivative.

11. A temperature distribution is given by:

    $$f(x,y) = 10 + 6\cos(x)\cos(y) + 3\cos(2x) + 4\cos(3y)$$

    Find the maximum directional derivative at point $P = \left(\frac{\pi}{3}, \frac{\pi}{3}\right)$

12. Let $f(x,y) = 4xy + 3y^2$

    a) Find the directional derivative of f in the direction of u=(2, -1) at point P=(1, 1)

    b) Find the maximum directional derivative.

13. Let take a differentiable function f defined on a open set U. Let assume that P is a point of U such that *f*(P) is the maximum of f, which means:

    $$\forall X \in U; \quad f(P) \geq f(X)$$

    Show that $gradf(P) = 0$