UNIVERSITE LIBANAISE — ANALYSE II — mars 09
FAC. DE GENIE II — T.D. 4 — WISSAM KARAM

1. Find the potential functions relative to the following field vectors:

   $(a)\ (4xy, 2x^2)$; $(b)\ (xy\cos xy + \sin xy, x^2 \cos xy)$

2. Find the potential function φ (x, y) associated to the field vector:

   $f(x,y) = (3x^2 y + 2y^2, x^3 + 4xy - 1)$ and verifying $\varphi(1,1) = 4$

3. What is the gradient of $f(x,y) = Arctg\left(\frac{y}{x}\right)$, defined over a rectangle that does not contain the line x=0

4. Find the potential function φ associated to the following field vector:

   $e^{y+2z}(1, x, 2x)$ with φ(1,1,1)=4.

5. Let's consider the differential form:

   $$\omega = \frac{dx}{\sqrt{x^2+y^2}} + \frac{\sqrt{x^2+y^2} - x}{y\sqrt{x^2+y^2}}dy$$

   defined on an open set of R^2 that does not contain (0,0)

   a) Show using polar coordinates that ω is a full differential of a function $U(r,\theta)$

   b) Give the equation of the curves U = Constant

   (Partiel 96 30 points)

6. f, g are 2 numerical functions of class C^1 defined over R. Find f and g in such a way that the differential form:

   $U(x,y,z) = 2xzdx + f(y)g(z)dy + (x^2 + y^2)dz$
   is exact, then find a function F(x, y, z) such that U = dF.

7. Let's consider

   $$P = e^{-x}\left[\frac{1}{x+y} - \ln(x+y)\right]; \qquad Q = \frac{e^{-x}}{x+y}$$

   Show that the field vector (P, Q) is the gradient of a function $\varphi(x,y)$ to be determined.

8. The following differential forms $\omega(x,y)$ are nit closed. Find $\tau(x,y)$ such that $\tau(x,y) \times \omega(x,y)$ is a closed form. Compute then the primitive of this product:

   a) $w(x,y) = (\cos(x+y) + \sin(x+y))dx + \cos(x+y)dy$ $\tau(x,y)$ only dependent of x.

   b) $w(x,y) = y(1-xy)dx + (y-x)dy$ $\tau(x,y)$ only dependent of y.