1. Evaluate the curve integrals of the following functions over the indicated paths :

   a) $F(x,y)=(y^2,-x)$, $x=\dfrac{y^2}{4}$ from (0,0) to (1,2)

   b) $F(x,y)=(y^2-x^2,x)$, over the arc $x^2+y^2=4$ from (0,2) to (2,0) clockwise.

   c) $F(x,y)=(y^2x^2,xy^2)$, over the closed path composed from the line x=1 and the parabola arc $x=y^2$ in the trigonometric direction.

   d) $F(x,y)=(x^2-2xy, y^2-2xy)$, $y=x^2$ from (-2,4) to (1,1)

   e) F(x, y, z)= (x, y, xz- y), along the segment (0, 0, 0), (1, 2, 4)

2. Same question for the field:

$$G(x,y)=\left(\frac{-y}{x^2+y^2},\frac{x}{x^2+y^2}\right)$$

   a) In the trigonometric direction along the circle $x^2+y^2=2$ from (1,1) to $(\sqrt{2},0)$.

   b) All over the circle.

   c) All over the following circle $x^2+y^2=1$

   d) Verify that: $\dfrac{\partial f}{\partial y}=\dfrac{\partial g}{\partial x}$

3. Evaluate the curve integral of (x y, y) along $x=2y^2$ from (2, -1) to (8, 2).

4. Same question for (2xy,-3xy) along the square composed by: x =3, x =5, y=1, y=3.

5. Compute the integral of the field F(x, y, z) = (2x, 3y, 4z) along the line C(t) = (t, t, t) between (0,0,0) and (1, 1, 1).

6. Same question for the field F(x, y, z) = (y+ z, x+ z, x+ y).

7. Evaluate the integral of the field here above along $C(t)=(t,t^2,t^4)$. What can we deduce?

8. Let P, Q 2 points in a 3-space. Show that the integral of the field $F(x,y,z)=(z^2,2y,2xz)$ between P and Q is independent of the path.

9. Let $F(x,y)=(\frac{x}{r^3},\frac{y}{r^3})$ where $r=\sqrt{x^2+y^2}$ Evaluate the integral of F along $C(t)=(e^t\cos t, e^t\sin t)$ between (1,0) and $(e^{2\pi},0)$.

10. Evaluate the integral of the field:

$$F(x,y)=\left(\frac{x\cos r}{r},\frac{y\cos r}{r}\right)$$

along:

a) The circle of unit radius, centered at the origin, between (1,0) and (0,1) counterclockwise.

b) The whole circle. What can we say?

11. Let $F(x,y)=\left(\dfrac{x-y}{x^2+y^2},\dfrac{x+y}{x^2+y^2}\right)$

a) Evaluate the integral of this field along the circle of unit radius, centered at the origin, counterclockwise.

b) Is this field a potential function?.

12. Let $F(x,y)=\left(\dfrac{-y+3x}{x^2+y^2},\dfrac{x+3y}{x^2+y^2}\right)$

a) Does F(x, y) have a potential function in the rectangle defined by: $1 \le x \le 2$, et $1 \le y \le 2$?. Why?.

b) Find the integral of F over the circle of unit radius, centered at the origin, counterclockwise.

13. Let the 4 points, A(2,0); B(1,1); C(1,0); D(0,-1). Γ is the combination of the arc AB of the circle centered at C and the segments BD, DO, OA. Evaluate the curve integral:

a) $\int_\Gamma (x^4 - x^3 e^x - y)dx + (x - y\,Arctg y)dy.$

Hint: A part of the function inside the integral has a potential function.

14. Let F(x, y) = (P(x, y), Q(x, y)) such that:

$$P(x,y)=x\ln(x^2+y^2)-y;\quad Q(0,y)=y\ln y^2$$

Find Q(x, y) so that F(x, y) is a gradient field. Deduce the curve integral of F(x, y) over a curve joining point (1, 1) to point (x, y)

15. Evaluate the integral of $w(x,y)=x^2dy+y^2dx$ over:

The ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}+\dfrac{2x}{a}$