1. Evaluate the following repeated integrals
   a) $\iint_D xydxdy$ where D is a set of points (x,y) such that: x ≥ 0, y ≥ 0, x+ y ≤ 1 (sol. $\frac{1}{24}$)
   b) $\iint_D (x+y)dxdy$ where D is a triangle formed by: O(0.0), A(1,1), B(2,0) (sol. $\frac{4}{3}$)
   c) $\iint_D xydxdy$ where D is defined by: $x \geq 0,\ y \geq 0,\ \frac{x^2}{a^2}+\frac{y^2}{b^2}-1 \leq 0$
   (sol. $\frac{\pi ab}{8}$)
   d) $\iint_D \frac{dxdy}{(x+y)^2}$ where D is defined by: x ≥ 1, y ≥ 1, x+y ≤ 3. (sol. $\ln\left(\frac{3}{2}\right)-\frac{1}{3}$)
2. Evaluate the following repeated integrals
   a) $\iint_D |x+y|dxdy$
   - where D is defined by: 0 < x < 2 et -2 < y < 2. (sol. $\frac{32}{3}$)
   - where D is defined by: $|x|<1,\quad |y|<1$ (sol. $\frac{8}{3}$)

   b) $\iint_D \frac{dxdy}{1+x^2+y^2}$ where D is the open disc $x^2+y^2<1$. (sol. $\pi \ln 2$)
3. Let D be the set of points (x,y) such that (x, y) verify :
   $x^2+y^2-y>0; x^2+y^2-x<0;$ y>0
   Evaluate $\iint_D (x+y)^2 dxdy$ (rép. $\frac{3}{16}$)
4. Evaluate the integral $\iint_D (x+y)dxdy \quad D=\{(x,y)\in R^{2+} / x+y<1\}$
   a) directly (sol. $\frac{1}{3}$)
   b) using level curves.
5. Evaluate the integral
   $\iint_D \frac{(2+x)y}{x^2+y^2}dxdy; D=\{(x,y)\in R^2 / y>0; y^2-2x<0;\ 8<x^2+y^2<48\}$
   (sol $8\sqrt{3}-4\sqrt{2}-4+\ln 2$)
6. Evaluate using a repeated integral the volume of the domain bounded by the surfaces:
   $x^2+(y-a)^2=a^2$; $x^2+y^2=4az$; z = 0. (sol. $\frac{3}{8}\pi a^3$)
7. Let P the pave: 0 ≤ x ≤ 1, 0 ≤ y ≤ 1. Evaluate over P the repeated integral of
   $f(x,y)=\frac{1}{(1+x+y)^2}$ (sol. $\ln\frac{4}{3}$)

8. Evaluate the repeated integral of:

   a) e^{x+y} where D is defined by: $|x|+|y| \le 1$ (sol. $e-\frac{1}{e}$)

   b) $x^2 - y^2$ where D is defined by the curve y = sin x pour $0 \le x \le \pi$. (sol. $\pi^2 - \frac{40}{9}$)

   c) $x^2 - y^2$ where D is defined by: $0 \le x \le 1$ et $x^2 - y^2 \ge 0$. (sol. $\frac{1}{3}$)

9. Evaluate the repeated integral of $f(x,y)=\frac{1}{r^n}$, where $r=\sqrt{x^2+y^2}$ *et* $n \in N^*$ where D is the domain bounded by 2 circles centered at the origin and having respectively a and b for radius, 0 < a < b.

   (sol. $n=1; 2(b-a)\pi;\ n=2,\ 2\pi(\ln b - \ln a);\ n=n,\ 2\pi\frac{a^{2-n}-b^{2-n}}{n-2}$)

10. Evaluate the cylindrical volume having a disc of radius 1 as a cross section and bounded form above and beneath by the sphere of radius 2. (sol. $\frac{4\pi}{3}\left(2\sqrt{2}-1\right)$)

11. Let $(a,b) \in R^2$

$$f(a,b) = \iint_D |x-y|dxdy \; D = \{(x,y) \in R^2 \,/\, 0 \le x \le a;\ 0 \le y \le b\}$$ evaluate f(a, b)

(sol. $\frac{a^3}{3} - \frac{a^2 b}{2} + \frac{ab^2}{2}$)

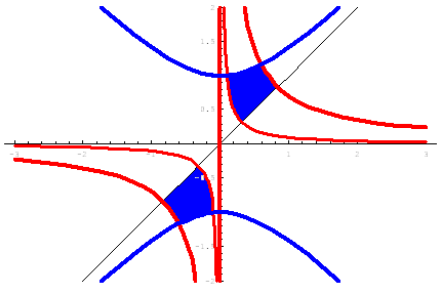
12. Let a and b, two real numbers such that: 0 < a < b.

Let: $D = \left\{(x,y) \in R^2 \,/\, a \le xy \le b; x \le y; y^2 - x^2 \le 1\right\}$

We perform the following variable substitution: $u = xy, v = y^2 - x^2$. We assume that the Jacobean product:

$$\frac{D(x,y)}{D(u,v)} \times \frac{D(u,v)}{D(x,y)} = 1$$

Evaluate: $\iint_D (y^2 - x^2)(y^2 + x^2)dxdy$ (sol. $\frac{b-a}{2}$)



Exercise 12

Additional problems

Find the moments of inertia I_x, I_y, I_O for the following areas:

a. The semi-circle of radius r, centered at the origin O and situated at the right side of Oy.

b. The isosceles triangle having the following summits $(0,0),(h,\frac{a}{2}),(h,-\frac{a}{2})$

c. The area between the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ and the circle $x^2+y^2=2y$