1. Use Green's theorem to evaluate:

$$\int_{C^+} y^2dx + xdy$$

where C is :

a) the curve linking the four summits: (0,0), (2,0), (2,2), (0,2).
b) the circle of unit radius, centered at the origin.

2. Use Green's theorem to evaluate:

$$\int_{C^+} y^2dx - xdy$$

Where C is the triangle having the following summits: (0,0), (0,1), (1,0).

$$\boxed{sol = -\frac{5}{6}}$$

3. Let C be the curve defined by: y = sin x and y = sin 2x, for 0 < x< π/3, oriented counterclockwise. Evaluate:

$$\int_{C} (1+y^2)dx + ydy$$

a) directly

b) using Green's theorem. $\boxed{sol = -\frac{3\sqrt{3}}{16}}$

4. Evaluate:

$$\int_{C} ydx + x^2dy$$

over the following paths:

a) The lines joining the points: (1,0), (0,1), (-1,0), (0,-1) $\boxed{sol = -2}$

b) The lines joining the points: (0,0),(1,1),(1,0) $\boxed{sol = \frac{1}{6}}$

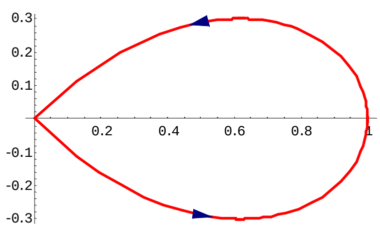
5. Let C be a closed curve, oriented counterclockwise. Let A be its interior. Show that:

a) $area(A) = \frac{1}{2}\int_{C^+} -ydx+xdy$

b) $area(A) = \int_{C^+} xdy$

c) if (C) is given in polar coordinates by $r = r(\theta)$ then:

$$area(A) = \frac{1}{2}\int_{(C)} r^2(\theta)d\theta$$



d) Deduce the area of : $r(\theta) = \frac{\cos 2\theta}{\cos\theta}$ with $-\frac{\pi}{4} \le \theta \le +\frac{\pi}{4}$

6. Let f be a harmonic function (Laplacien=0). Show that if A is the interior of a closed curve then: $\int_{C} \frac{\partial f}{\partial y}dx - \frac{\partial f}{\partial x}dy = 0$

7. Evaluate the following integral:

$$\int_C \frac{-y}{x^2+y^2}dx + \frac{x}{x^2+y^2}dy$$

Where C is one of the following paths:

a) The curve $y^2 = 2(x+2)$ and $x = 2$ oriented counterclockwise.

b) The square having the following summits: (1.0), (0,1), (-1,0), (0,-1), oriented counterclockwise. $\boxed{sol = 2\pi}$

8. Evaluate this fields integral:

$$F(x,y) = \left(\frac{-y+x}{x^2+y^2}, \frac{x+y}{x^2+y^2} \right)$$

Around the curves given in exercise 7.

9. Let F(x,y)=(y,-x). Let C be a circle of unit radius centered at the origin oriented counterclockwise. Show that:

$$\int_C \mathrm{F.nds} = 0$$

10. Evaluate the volume delimited by the plane (xOy), and the surfaces:

$$2z = \frac{x^2}{p} + \frac{y^2}{q} \quad p > 0, q > 0 \ \ and \ \ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

a) Using a direct repeated integral.

b) Using Green's theorem. $\boxed{sol = \dfrac{ab\pi(b^2p + a^2q)}{8pq}}$

11. Evaluate the curve integral:

$$\int_C (2xy - x^2)dx + (x + y^2)dy$$

Where C is the closed curve defined by: $y = x^2 \ et \ x = y^2$.

a) Directly.

b) Using Green's theorem. $\boxed{sol = \dfrac{1}{30}}$

12. Let: $I(n) = \int_0^{\pi/4} \cos^n \theta \, d\theta$; $J(n) = \int_0^{\pi/4} \sin^n \theta \, d\theta$

a) Prove that: $nI(n) = \left(\frac{\sqrt{2}}{2}\right)^n + (n\text{-}1)I(n\text{-}2)$ et $nJ(n) = -\left(\frac{\sqrt{2}}{2}\right)^n + (n\text{-}1)J(n\text{-}2)$

Deduce the values of I(n) and J(n) when n = {0, 2, 4, 6, 8}.

b) Let D be the domain defined by: $x^2 + y^2 - x \le 0, \ \ x^2 + y^2 - y \le 0, \ \ y \le 0$

Evaluate $\mathrm{I} = \iint_D (x^2 + xy)dxdy$

c) Evaluate the curve integral: $J = \int_C \frac{-xy^2}{2}dx + \frac{x^3}{3}dy$ around the curve C delimiting the previous domain D.