1. Evaluate the area of the following surfaces:
   a) A cone of height h, obtained by turning the line $z = ax$ around the z axis (z ≥0).
   sol. $\frac{\pi h^2}{a^2}\sqrt{1+a^2}$
   b) The part of z =x y located above the disc $x^2 + y^2 \leq 1$. **sol**. $\frac{2\pi}{3}\left(2\sqrt{2}-1\right)$
   c) The part of the sphere $x^2 + y^2 + z^2 = 1$ located between the planes
   $z = \frac{1}{\sqrt{2}}$ et z = -$\frac{1}{\sqrt{2}}$. **sol**. $2\sqrt{2}\pi$

2. Integrate the following functions on the indicated surfaces:
   a) $x^2 + y^2$ on the hemisphere of radius *a* centered at the origin. **sol**.: $\frac{4\pi a^4}{3}$
   b) $(x^2 + y^2)^2 z^2$ on the hemisphere of radius *a* centered at the origin. **sol**.: $\frac{16a^8\pi}{105}$
   c) z^2 on $x^2 + y^2 + z^2 = 1$ **sol**.: $\frac{4\pi}{3}$
   d) z on $z = x^2 + y^2$ with $x^2 + y^2 \leq 1$ **sol**.: $\frac{\pi}{60}(1+25\sqrt{5})$
   e) z on $z = 1 - x^2 - y^2$ with $0 \leq z \leq 1$. **sol**.: $\frac{\pi}{60}(25\sqrt{5}-11)$
   f) x on $x^2 + y^2 = z^2$ with $0 \leq z \leq a$. **sol**.: 0
   g) x^2 on $x^2 + y^2 = a^2$ with $0 < z < a$. **sol**.: πa^4

3. Integrate the following vector fields on the indicated surfaces (in other terms: evaluate their flux):
   a) $F(x, y, z) = \frac{1}{\sqrt{x^2 + y^2}}(y, -y, 1)$ $z = 1 - x^2 - y^2$ with $0 \leq z \leq 1$ **sol**. $\frac{4\pi}{3}$
   b) F(*x, y, z*) = (*y, -x, 1*) on $X(r, \theta) = (r\cos\theta, r\sin\theta, \theta)$ with 0 ≤ r ≤1 et 0 ≤θ ≤2π.
   sol 2π
   c) $F(x, y, z) = (x^2, y^2, z^2)$ on *X*(*t, u*) = (*t* +*u, t- u, t*) with 0 ≤ t ≤ 2 et 1 ≤ u ≤ 3.
   sol. $\frac{104}{3}$
   d) F(*x, y, z*) = (*x*, 0, 0) on the part of the unit sphere, centered at origin and located inside the cone $z^2 = x^2 + y^2$; z ≥ 0. **sol**. $\frac{\pi}{12}(8-5\sqrt{2})$

4. Let S be the boundary of the unit cube: 0 ≤ *x* ≤ 1, 0 ≤ *y* ≤ 1, 0 ≤ *z* ≤ 1. Evaluate on this cube the integral of the vector field $F(x, y, z) = (xy, y^2, y^2)$ **Sol.** $\frac{3}{2}$

5. Evaluate the area of the part of the cone $x^2 + y^2 = 3z^2$ located inside the surface $x^2 + y^2 = 4z$ and such that z ≥0. **sol.** $\frac{32\pi\sqrt{3}}{9}$

6. Evaluate the integral $\iint_S rotF.n\, d\sigma$ where F is the vector field:
   $F(x, y, z) = (-y, x^2, z^3)$ and S the surface $x^2 + y^2 + z^2 = 1$; $-\frac{1}{2} \leq z \leq 1$ **sol** $\frac{3\pi}{4}$

7. Verify the Stokes theorem for: $F(x,y,z)=(x^2+y, y\,z, x-z^2)$ where S is the triangle $2x+y+2z=2,\quad x\geq 0; y\geq 0;\ z\geq 0$ **sol.** $-\dfrac{13}{6}$

8. Evaluate using the Stokes theorem: $\iint_S$ rotF.n dσ

   a) $F(x,y,z)=(x,y,z)$ on the triangle that's summits are (1,0,0,), (0,1,0), (0,0,1). **Sol.** 0

   b) $F(x,y,z)=(x+y, y-z, x+y+z)$ on the hemisphere $x^2+y^2+z^2=a^2$, z ≥0.
   sol. $-a^2\pi$

9. Use the divergence theorem to evaluate the following integral:
   $\iint_S xz^2dydz+(x^2y-z^3)dzdx+(2xy-y^2z)dxdy$ where S is the surface composed by the hemisphere $z=\sqrt{a^2-x^2-y^2}$ and the plane z = 0. **sol.** $\dfrac{2\pi a^5}{15}$

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10. Let S be a part of the surface bounded by the closed curve C. We shall denote by α , β, Γ the cosines of the normal to S directed toward the exterior.
    a) Transform the curve integral into a surface integral:
    $I=\iint_S [\alpha x(z^2-y^2)+\beta y(x^2-z^2)+\gamma z(y^2-x^2)]d\sigma$
    b) Evaluate I when S is a part of the plane x=1 defined by:
    $y^2+z^2\leq 1,\ z+y\geq 0,\ z-y\geq 0$

(Sciences 82-83, 26 pts)

11. Let a closed surface S. D it's interior, n the normal unit vector directed toward the exterior of S. Prove that the volume of D is given by:
    $V=\iint_S xdydz=\frac{1}{3}\iint_S xdydz+ydzdx+zdxdy$

(Sciences 82-83, 10 pts)

12. Show that the vector field $F(x,y,z)=(x^2-yz, y^2-xz, z^2-xy)$ is the gradient of a scalar field f that we shall find. Evaluate using Ostrogradsky theorem the flux of the field exiting the sphere:
    $x^2+y^2+z^2-2ax-2ay-2az+2a^2=0$

(Sciences 79-80, 24 pts)

13. Let's consider the conical surface having the following equation: $x^2+y^2=z^2$, z ≥0.

    a) Show that the normal vector to this surface is given by $(\frac{x}{z},\frac{y}{z},-1)$

    b) Evaluate the flux of the vector field: V = (2x, -2y, z²), through the surface S bounded by the cone $x^2+y^2=z^2$; z ≥ 0 and the disc $x^2+y^2\leq 4$ with z = 2, the normal vector being directed to the exterior.

    c) Deduce the value of: $I=\iiint_D zdxdydz$ where D is the domain bounded by the surface S.

(Sciences 84-85, 25 pts)