TD3 Derivée d'une fonction composée

1. Soit f une fonction vérifiant $gradf\,(1{,}1{,}1) = (5{,}2{,}1)$. Soit $C(t) = (t^2, t^{-3}, t)$

   Calculer $f_t^{'}(C(t))$ en $t = 1$

Corrigé 1

$$f_t^{'}(C(t)) = gradf\,(C(t)).C'(t)$$

$$\Rightarrow C'(t) = (2t, -3t^{-4}, 1) \Rightarrow C'(1) = (2, -3, 1)$$

$$\Rightarrow \quad f_t^{'}(C(1)) = gradf\,(C(1)).C'(1) = (5,2,1).(2,-3,1) = 10 - 6 + 1 = 5$$

---

2. Considérons les coordonnées polaires: x=rcosθ y= rsinθ .

   a) Calculer: $r_x^{'}, \; r_y^{'}, \; \theta_x^{'}, \; \theta_y^{'}$

   b) Montrer en utilisant un changement en coordonnées polaires que toute fonction dérivable f des deux variables x et y, vérifiant l'identité:

   $$xf_x^{'} + yf_y^{'} = 0$$

   se réduit à une fonction de $\dfrac{y}{x}$.

Corrigé 2

- $x = r\cos\theta, \; y = r\sin\theta \; \Rightarrow r = \sqrt{x^2 + y^2}, \; \theta = ArcTan\left(\dfrac{y}{x}\right)$
- $r_x^{'} = \dfrac{x}{r}; \; r_y^{'} = \dfrac{y}{r} \; ; \; \theta_x^{'} = \dfrac{-y}{x^2 + y^2} \; ; \; \theta_y^{'} = \dfrac{x}{x^2 + y^2}$
- Considérons $f\left(r(x,y), \theta(x,y)\right)$
  - $f_x^{'} = f_r^{'}.r_x^{'} + f_\theta^{'}.\theta_x^{'} = \dfrac{x}{r} f_r^{'} - \dfrac{y}{x^2 + y^2} f_\theta^{'}$
  - $f_y^{'} = f_r^{'}.r_y^{'} + f_\theta^{'}.\theta_y^{'} = \dfrac{y}{r} f_r^{'} + \dfrac{x}{x^2 + y^2} f_\theta^{'}$
  - $xf_x^{'} + yf_y^{'} = \dfrac{x^2}{r} f_r^{'} - \dfrac{xy}{x^2 + y^2} f_\theta^{'} + \dfrac{y^2}{r} f_r^{'} + \dfrac{xy}{x^2 + y^2} f_\theta^{'}.$
  - $xf_x^{'} + yf_y^{'} = \dfrac{x^2 + y^2}{r} f_r^{'} = rf_r^{'} = 0 \Rightarrow f_r^{'} = 0$

f est fonction de θ seulement c'est-à-dire $f\left(\dfrac{y}{x}\right)$

---

3. Si U = *f(x- y, y- x)*, Montrer que: $\frac{\partial U}{\partial x}+\frac{\partial U}{\partial y}=0$

Corrigé 3

- Posons $v = x - y$ et $w = y - x$
- $\frac{\partial U}{\partial x}+\frac{\partial U}{\partial y}=\frac{\partial f}{\partial v}\frac{\partial v}{\partial x}+\frac{\partial f}{\partial w}\frac{\partial w}{\partial x}+\frac{\partial f}{\partial v}\frac{\partial v}{\partial y}+\frac{\partial f}{\partial w}\frac{\partial w}{\partial y}$
- $\frac{\partial U}{\partial x}+\frac{\partial U}{\partial y}=\frac{\partial f}{\partial v}-\frac{\partial f}{\partial w}-\frac{\partial f}{\partial v}+\frac{\partial f}{\partial w}=0$

---

4. Soit la fonction U définie par $U(x,\ y,\ z)=F\left(\frac{y-x}{xy},\frac{z-x}{xz}\right)$ Montrer que : $x^2U'_x+y^2U'_y+z^2U'_z=0$

Corrigé 4

- Posons $v=\frac{y-x}{xy}$ et $w=\frac{z-x}{xz}$
- $v'_x=\frac{-xy-(y-x)y}{x^2y^2}=-\frac{1}{x^2}$; $v'_y=\frac{xy-(y-x)x}{x^2y^2}=\frac{1}{y^2}$; $v'_z=0$
- $w'_x=\frac{-xz-(z-x)z}{x^2z^2}=-\frac{1}{x^2}$; $w'_y=0$; $w'_z=\frac{xz-(z-x)x}{x^2z^2}=\frac{1}{z^2}$
- $x^2U'_x+y^2U'_y+z^2U'_z=x^2\left(U'_vv'_x+U'_ww'_x\right)+y^2\left(U'_vv'_y+U'_ww'_y\right)+z^2\left(U'_vv'_z+U'_ww'_z\right)$
- $x^2U'_x+y^2U'_y+z^2U'_z=x^2\left(-\frac{1}{x^2}U'_v-\frac{1}{x^2}U'_w\right)+y^2\left(\frac{1}{y^2}U'_v\right)+z^2\left(\frac{1}{z^2}U'_w\right)$
- $x^2U'_x+y^2U'_y+z^2U'_z=-U'_v-U'_w+U'_v+U'_w=0$

---

5. Soit *g(x, y)= f(x+ y, x- y)*, où f est une fonction différentiable de deux variables u et v. Montrer que:

$$\frac{\partial g}{\partial x}\frac{\partial g}{\partial y}=\left(\frac{\partial f}{\partial u}\right)^2-\left(\frac{\partial f}{\partial v}\right)^2$$

Corrigé 5

- Posons $u = x + y$ et $v = x - y$
- $\frac{\partial g}{\partial x}\frac{\partial g}{\partial y}=\left(\frac{\partial f}{\partial u}\frac{\partial u}{\partial x}+\frac{\partial f}{\partial v}\frac{\partial v}{\partial x}\right)\left(\frac{\partial f}{\partial u}\frac{\partial u}{\partial y}+\frac{\partial f}{\partial v}\frac{\partial v}{\partial y}\right)$

- $\frac{\partial g}{\partial x}\frac{\partial g}{\partial y}=\left(\frac{\partial f}{\partial u}+\frac{\partial f}{\partial v}\right)\left(\frac{\partial f}{\partial u}-\frac{\partial f}{\partial v}\right)$
- $\frac{\partial g}{\partial x}\frac{\partial g}{\partial y}=\left(\frac{\partial f}{\partial u}\right)^2-\left(\frac{\partial f}{\partial v}\right)^2$

---

6. A l'aide du changement de variable défini par $u=xy$ et $v=x/y$, déterminer les fonctions $z(x, y)$ satisfaisant : $xz'_x + yz'_y = 2xy$

(Final 87-88, 6pts)

Corrigé 6

- $u = xy \Rightarrow u'_x = y;\quad u'_y = x;$
- $v=\frac{x}{y}\Rightarrow v'_x=\frac{1}{y};\quad v'_y=-\frac{x}{y^2}$
- $z'_x = z'_u u'_x + z'_v v'_x = yz'_u + \frac{1}{y}z'_v$
- $z'_y = z'_u u'_y + z'_v v'_y = xz'_u - \frac{x}{y^2}z'_v$
- $xz'_x + yz'_y = xyz'_u + \frac{x}{y}z'_v + xyz'_u - \frac{x}{y}z'_v = 2xyz'_u = 2xy$
- $2xyz'_u = 2xy \Rightarrow z'_u = 1 \Rightarrow z = u+\alpha(v) \Rightarrow z = xy+\alpha\left(\frac{x}{y}\right)$

---

7. Déterminer toutes les fonctions de classe C^2 de R_*^{2+} dans R_*^{2+} telles que:

$$x^2\frac{\partial^2 f}{\partial x^2}-y^2\frac{\partial^2 f}{\partial y^2}=0$$

On pourra utiliser le changement de variables $v= x/y$; $u= xy$

(Final 88-89,10 pts)

Corrigé 7

- $u = xy \Rightarrow u'_x = y;\quad u''_{x^2}=0;\quad u'_y = x;\quad u''_{y^2}=0\ ;$
- $v=\frac{x}{y}\Rightarrow v'_x=\frac{1}{y}, v''_{x^2}=0;\ v'_y=-\frac{x}{y^2}; v''_{y^2}=\frac{2x}{y^3}$
- Posons : $f(x,y)=F[u(x,y),v(x,y)]$
- $\frac{\partial f}{\partial x}=F'_u.u'_x+F'_v.v'_x \Rightarrow \frac{\partial f}{\partial x}=yF'_u+\frac{1}{y}F'_v$
  - o $\frac{\partial^2 f}{\partial x^2}=y\left[F''_{u^2}u'_x+F''_{uv}v'_x\right]+\frac{1}{y}\left[F''_{vu}u'_x+F''_{v^2}v'_x\right]$

- $\frac{\partial^2 f}{\partial x^2} = y\left[F''_{u^2} y + F''_{uv}\frac{1}{y}\right] + \left[F''_{vu} y + F''_{v^2}\frac{1}{y}\right].\frac{1}{y}$
- $\frac{\partial^2 f}{\partial x^2} = F''_{u^2} y^2 + F''_{uv} + F''_{vu} + F''_{v^2}\frac{1}{y^2}$

D'après le théorème de Schwartz puisque f est de classe $C^2 \Rightarrow F''_{vu} = F''_{uv}$

- $x^2\frac{\partial^2 f}{\partial x^2} = x^2 y^2 F''_{u^2} + 2x^2 F''_{uv} + \frac{x^2}{y^2} F''_{v^2}$

* $\frac{\partial f}{\partial y} = F'_u.u'_y + F'_v.v'_y \Rightarrow \frac{\partial f}{\partial y} = xF'_u - \frac{x}{y^2}F'_v$
  - $\frac{\partial^2 f}{\partial y^2} = x[F''_{u^2}u'_y + F''_{uv}v'_y] - \frac{x}{y^2}[F''_{vu}u'_y + F''_{v^2}v'_y] + \frac{2xy}{y^4}F'_v$
  - $\frac{\partial^2 f}{\partial y^2} = x\left[F''_{u^2}x - \frac{x}{y^2}F''_{uv}\right] - \frac{x}{y^2}\left[F''_{vu}x - \frac{x}{y^2}F''_{v^2}\right] + F'_v\frac{2xy}{y^4}$
  - $y^2\frac{\partial^2 f}{\partial y^2} = F''_{u^2}x^2y^2 - 2x^2F''_{uv} + \frac{x^2}{y^2}F''_{v^2} + F'_v\frac{2xy}{y^2}$
* $x^2\frac{\partial^2 f}{\partial x^2} - y^2\frac{\partial^2 f}{\partial y^2} = 4x^2F''_{uv} - \frac{2x}{y}F'_v = 4uvF''_{uv} - 2vF'_v = 0$
  - $4uvF''_{vu} - 2vF'_v = 0 \Rightarrow 2uF''_{vu} = F'_v$
  - Posons $g = F'_v$
  - $2ug'_u = g \Rightarrow \frac{g'_u}{g} = \frac{1}{2u} \Rightarrow Log(g) = Log\left(\alpha(u)\sqrt{u}\right) \Rightarrow g = \alpha(v)\sqrt{u}$
  - $F'_v = \alpha(v)\sqrt{u} \Rightarrow F(u,v) = \sqrt{u}\int\alpha(v)dv = \beta(v)\sqrt{u} + C^{te}$

---

8. Le Laplacien dune fonction est défini par $\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$. Une fonction est dite harmonique si son Laplacien est nul

a) Montrer que le Laplacien d'une fonction à deux variables s'écrit en coordonnées polaires:

$$\Delta u = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}$$

b) Trouver la fonction harmonique la plus générale quand u est fonction de r seulement.

c) Même question si $u = r^n f(\theta)$, n étant un entier naturel.

(Partiel 87-88, 28pts)

Corrigé 8

- $x = r\cos\theta\,,\quad y = r\sin\theta\ ;$
- $r'_x = \dfrac{x}{r};\qquad r'_y = \dfrac{y}{r}$
- $\theta'_x = \dfrac{-y}{x^2+y^2} = -\dfrac{y}{r^2}\ ;\qquad \theta'_y = \dfrac{x}{x^2+y^2} = \dfrac{x}{r^2}$

- $\dfrac{\partial f}{\partial x} = \dfrac{\partial f}{\partial r}\dfrac{\partial r}{\partial x} + \dfrac{\partial f}{\partial \theta}\dfrac{\partial \theta}{\partial x} = \dfrac{x}{r}\dfrac{\partial f}{\partial r} - \dfrac{y}{r^2}\dfrac{\partial f}{\partial \theta}$
- $\dfrac{\partial^2 f}{\partial x^2} = \dfrac{\partial}{\partial x}\left(\dfrac{x}{r}\dfrac{\partial f}{\partial r} - \dfrac{y}{r^2}\dfrac{\partial f}{\partial \theta}\right) = \dfrac{\partial}{\partial x}\left(\dfrac{x}{r}\dfrac{\partial f}{\partial r}\right) - \dfrac{\partial}{\partial x}\left(\dfrac{y}{r^2}\dfrac{\partial f}{\partial \theta}\right)$
- $$\boxed{\begin{aligned}\frac{\partial}{\partial x}\left(\frac{x}{r}\frac{\partial f}{\partial r}\right) &= \frac{\partial}{\partial x}\left(\frac{x}{r}\right)\frac{\partial f}{\partial r} + \frac{x}{r}\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial r}\right)\\ &= \frac{r - x\frac{x}{r}}{r^2}\frac{\partial f}{\partial r} + \frac{x}{r}\left(\frac{\partial^2 f}{\partial r^2}\frac{\partial r}{\partial x} + \frac{\partial^2 f}{\partial \theta \partial r}\frac{\partial \theta}{\partial x}\right)\\ &= \frac{y^2}{r^3}\frac{\partial f}{\partial r} + \frac{x^2}{r^2}\frac{\partial^2 f}{\partial r^2} - \frac{xy}{r^3}\frac{\partial^2 f}{\partial \theta \partial r}\end{aligned}}$$
- $$\boxed{\begin{aligned}\frac{\partial}{\partial x}\left(\frac{y}{r^2}\frac{\partial f}{\partial \theta}\right) &= \frac{\partial}{\partial x}\left(\frac{y}{r^2}\right)\frac{\partial f}{\partial \theta} + \frac{y}{r^2}\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial \theta}\right)\\ &= \frac{-2xy}{r^4}\frac{\partial f}{\partial \theta} + \frac{y}{r^2}\left(\frac{\partial^2 f}{\partial r \partial \theta}\frac{\partial r}{\partial x} + \frac{\partial^2 f}{\partial \theta^2}\frac{\partial \theta}{\partial x}\right)\\ &= \frac{-2xy}{r^4}\frac{\partial f}{\partial \theta} + \frac{xy}{r^3}\frac{\partial^2 f}{\partial r \partial \theta} - \frac{y^2}{r^4}\frac{\partial^2 f}{\partial \theta^2}\end{aligned}}$$
- $$\boxed{\frac{\partial^2 f}{\partial x^2} = \frac{y^2}{r^3}\frac{\partial f}{\partial r} + \frac{x^2}{r^2}\frac{\partial^2 f}{\partial r^2} - \frac{2xy}{r^3}\frac{\partial^2 f}{\partial \theta \partial r} + \frac{2xy}{r^4}\frac{\partial f}{\partial \theta} + \frac{y^2}{r^4}\frac{\partial^2 f}{\partial \theta^2}}$$
- $\dfrac{\partial f}{\partial y} = \dfrac{\partial f}{\partial r}\dfrac{\partial r}{\partial y} + \dfrac{\partial f}{\partial \theta}\dfrac{\partial \theta}{\partial y} = \dfrac{y}{r}\dfrac{\partial f}{\partial r} + \dfrac{x}{r^2}\dfrac{\partial f}{\partial \theta}$
- $\dfrac{\partial^2 f}{\partial y^2} = \dfrac{\partial}{\partial y}\left(\dfrac{y}{r}\dfrac{\partial f}{\partial r} + \dfrac{x}{r^2}\dfrac{\partial f}{\partial \theta}\right) = \dfrac{\partial}{\partial y}\left(\dfrac{y}{r}\dfrac{\partial f}{\partial r}\right) + \dfrac{\partial}{\partial y}\left(\dfrac{x}{r^2}\dfrac{\partial f}{\partial \theta}\right)$

- $$\begin{aligned}\frac{\partial}{\partial y}\left(\frac{y}{r}\frac{\partial f}{\partial r}\right) &= \frac{\partial}{\partial y}\left(\frac{y}{r}\right)\frac{\partial f}{\partial r}+\frac{y}{r}\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial r}\right)\\ &= \left(\frac{r-y\dfrac{y}{r}}{r^2}\right)\frac{\partial f}{\partial r}+\frac{y}{r}\left(\frac{\partial^2 f}{\partial r^2}\frac{\partial r}{\partial y}+\frac{\partial^2 f}{\partial\theta\partial r}\frac{\partial\theta}{\partial y}\right)\\ &= \frac{x^2}{r^3}\frac{\partial f}{\partial r}+\frac{y}{r}\left(\frac{\partial^2 f}{\partial r^2}\frac{y}{r}+\frac{\partial^2 f}{\partial\theta\partial r}\frac{x}{r^2}\right)\\ &= \frac{x^2}{r^3}\frac{\partial f}{\partial r}+\frac{y^2}{r^2}\frac{\partial^2 f}{\partial r^2}+\frac{xy}{r^3}\frac{\partial^2 f}{\partial\theta\partial r}\end{aligned}$$
- $$\begin{aligned}\frac{\partial}{\partial y}\left(\frac{x}{r^2}\frac{\partial f}{\partial\theta}\right) &= \frac{\partial}{\partial y}\left(\frac{x}{r^2}\right)\frac{\partial f}{\partial\theta}+\frac{x}{r^2}\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial\theta}\right)\\ &= \frac{-x\left(2r\dfrac{y}{r}\right)}{r^4}\frac{\partial f}{\partial\theta}+\frac{x}{r^2}\left(\frac{\partial^2 f}{\partial r\partial\theta}\frac{\partial r}{\partial y}+\frac{\partial^2 f}{\partial\theta^2}\frac{\partial\theta}{\partial y}\right)\\ &= \frac{-2xy}{r^4}\frac{\partial f}{\partial\theta}+\frac{xy}{r^3}\frac{\partial^2 f}{\partial r\partial\theta}+\frac{x^2}{r^4}\frac{\partial^2 f}{\partial\theta^2}\end{aligned}$$
- $$\frac{\partial^2 f}{\partial y^2}=\frac{x^2}{r^3}\frac{\partial f}{\partial r}+\frac{y^2}{r^2}\frac{\partial^2 f}{\partial r^2}+\frac{2xy}{r^3}\frac{\partial^2 f}{\partial\theta\partial r}-\frac{2xy}{r^4}\frac{\partial f}{\partial\theta}+\frac{x^2}{r^4}\frac{\partial^2 f}{\partial\theta^2}$$
- $$\frac{\partial^2 f}{\partial x^2}=\frac{y^2}{r^3}\frac{\partial f}{\partial r}+\frac{x^2}{r^2}\frac{\partial^2 f}{\partial r^2}-\frac{2xy}{r^3}\frac{\partial^2 f}{\partial\theta\partial r}+\frac{2xy}{r^4}\frac{\partial f}{\partial\theta}+\frac{y^2}{r^4}\frac{\partial^2 f}{\partial\theta^2}$$
- $$\Delta=\frac{\partial^2 f}{\partial r^2}+\frac{1}{r}\frac{\partial f}{\partial r}+\frac{1}{r^2}\frac{\partial^2 f}{\partial\theta^2}$$

TD 3 Plan Tangent – Dérivée directionnelle

1. Soit $f(x,y,z) = z - e^x \sin(y)$ et $P = \left(Log(3), \frac{3\pi}{2}, -3 \right)$. Calculer :

   a) La dérivée directionnelle de f au point P dans la direction A=(1, 2, 2)

   b) Le maximum et le minimum de la dérivée directionnelle de f au point P.

Corrigé 1

a) $u = \frac{\|A\|}{A} = \frac{1}{3}(1,2,2)$

$D_u f(P) = gradf(P).u$

$\Rightarrow gradf(X) = (-e^x \sin(y), e^x \cos(y), 1) \Rightarrow gradf(P) = (3,0,1)$

$\Rightarrow D_u f(P) = (3,0,1).\frac{1}{3}(1,2,2) = \frac{5}{3}$

b) $D_{\max} f(P) = \|gradf(P)\| = \sqrt{10} \quad \Rightarrow D_{\min} f(P) = -\sqrt{10}$

---

2. Trouver la dérivée directionnelle pour les données suivantes :

   a) $f(x,y) = Log\left(\sqrt{x^2+y^2}\right)$; P = (1,1) ; u = (2,1)

   b) $f(x,y.z) = xy + yz + zx$; P = (-1,1,7) ; u = (3,4,-12)

   c) $f(x,y.z) = 4x^2 + 9y^2$; P = (2,1) dans la direction de la dérivée directionnelle maximu,.

Corrigé 2

a) $f(x,y) = Log\left(\sqrt{x^2+y^2}\right)$; P = (1,1) ; u = (2,1)

$w = \frac{\|u\|}{u} = \frac{1}{\sqrt{5}}(2,1)$

$D_w f(P) = gradf(P).w$

$\Rightarrow gradf(X) = \left(\frac{x}{x^2+y^2}, \frac{y}{x^2+y^2} \right) \Rightarrow gradf(P) = \left(\frac{1}{2}, \frac{1}{2} \right)$

$\Rightarrow D_w f(P) = gradf(P).w = \left(\frac{1}{2}, \frac{1}{2} \right) \frac{1}{\sqrt{5}}(2,1). \frac{3}{2\sqrt{5}}$

b) $f(x,y.z)=xy+yz+zx$;   P = (-1,1,7) ; u = (3,4,-12)

$$w=\frac{\|u\|}{u}=\frac{1}{\sqrt{169}}(3,4,-12)=\frac{1}{13}(3,4,-12)$$

$$D_w f(P)=gradf(P).w$$

$$\Rightarrow gradf(X)=(y+z,x+z,x+y)\Rightarrow gradf(P)=(8,6,0)$$

$$\Rightarrow D_w f(P)=(8,6,0).\frac{1}{13}(3,4,-12)=\frac{48}{13}$$

c) $f(x,y.z)=4x^2+9y^2$;   P = (2,1)

$$D_{\max} f(P)=\|gradf(P)\|$$

$$\Rightarrow gradf(x,y)=(8x,18y)\Rightarrow gradf(P)=(16,18)$$

$$\Rightarrow D_w f(P)=\|(16,18)\|=2\sqrt{145}$$

---

3. Une distribution de la température est donnée par :

$$f(x,y)=10+6\cos(x)\cos(y)+3\cos(2x)+4\cos(3y)$$

Trouver la direction où la dérivée directionnelle est maximum au point :

$$P=\left(\frac{\pi}{3},\frac{\pi}{3}\right)$$

Corrigé 3

$$gradf(x,y)=(-6\sin(x)\cos(y)-6\sin(2x),-6\sin(y)\cos(x)-12\sin(3y))$$

$$gradf(P)=\left(-\frac{9\sqrt{3}}{2},-3\sqrt{3}\right)$$

---

4. Soit la fonction $f(x,y)=4xy+3y^2$

Corrigé 4

d) Trouver la dérivée directionnelle de f dans la direction de u=(2, -1) au point P=(1, 1)

$$w=\frac{\|u\|}{u}=\frac{1}{\sqrt{5}}(2,-1)$$

$$D_w f(P)=gradf(P).w$$

$$\Rightarrow gradf(X)=(4y,4x+6y)\Rightarrow gradf(P)=(4,10)$$

$$\Rightarrow D_w f(P)=-\frac{2}{\sqrt{5}}$$

e) Trouver la dérivée directionnelle maximale.

$$D_{\max} f(P) = \|gradf(P)\| = \sqrt{116}$$

---

5. Considérons une fonction f, différentiable dans un ensemble ouvert U. Supposons que P est un point de U tel que *f*(P) est le maximum.de *f* c'est-à-dire :

$$\forall X \in U; \quad f(P) \geq f(X)$$

Montrer que $gradf(P) = 0$

Corrigé 5

Si $\forall X \in U; \quad f(P) \geq f(X) \Rightarrow$ ceci est vrai pour C(t)=P+tA où un vecteur unitaire quelconque.

$\Rightarrow \forall t; \quad f(P) \geq f(P+tA)$

$\Rightarrow$ Comme $f(P+tA)$ est une courbe qui passe par $f(P)$ son maximum est atteint pour $f_t^{'}(P) = 0$

$\Rightarrow$ Or $f_t^{'}(P+tA) = gradf(P+tA).A \Rightarrow f_t^{'}(P) = gradf(P).A = 0$

$\Rightarrow$ Puisque ceci est vrai pout tout A

$\Rightarrow gradf(P) = 0$